# Problems on conformal kinematics

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`Mathematica` might be helpful in some of the exercises.

## 1 Weight-shifting operators

1. Given the construction of 1d embedding space from yesterday, i.e.

$$X^m = \gamma^m_{\alpha\beta}\chi^\alpha\chi^\beta, \tag{1}$$

   show that there exist weight-shifting operators transforming in spin-$j$ irrep of Spin$(2,1)$

$$\mathcal{D}_\mu^{(\alpha_1\ldots\alpha_{2j})} \tag{2}$$

   which change the scaling dimension by the weight $\mu = -j, -j+1\ldots, j$, and that there are no other weight-shifting operators. (Not using the classification from the lecture.)

2. Rewrite operators of $j = 1$ in terms of usual embedding space variable $X^m$ and check that they satisfy the consistency conditions, i.e.

$$\mathcal{D}_\mu^m X^2 f(X) \propto X^2. \tag{3}$$

3. (a) Write down tensor structures for three-point functions

$$\langle\phi_1\phi_2\phi_3\rangle, \quad \langle\phi_1\psi_2\psi_3\rangle, \tag{4}$$

   where $\phi_i$ are scalars and $\psi_i$ are fermions.

   (b) Define action of parity by[1]

$$\chi \to \gamma^2\chi, \tag{5}$$

[1] We use basis of gamma-matrices with

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

and show that this implies that the contractions

$$(\chi_1\chi_2) = \chi_{1,\alpha}\chi_2^\alpha \tag{6}$$

are parity-odd. Check that for each of the two correlation functions in (a) there is one parity-odd and one parity-even structure. We will label these structures by

$$\langle \cdots \rangle^{(\pm)}. \tag{7}$$

(c) It is convenient to describe $j = 1/2$ operators by contracting the free index with a "polarization" spinor $s$,

$$\mathcal{D}_\mu^\alpha \to s_\alpha \mathcal{D}_\mu^\alpha. \tag{8}$$

Show that if we act with parity also on $s$, then one of the two $j = 1/2$ weight-shifting operators is parity-odd and the other is parity-even.

(d) Consider the crossing equation

$$s_\alpha \langle \phi_1 \phi_2 (\mathcal{D}_\mu^\alpha \phi_3') \rangle^{(a)} = \sum_{\nu=\pm\frac{1}{2}, b=\pm} C_b^a(\mu,\nu) s_\alpha \langle (\mathcal{D}_\nu^\alpha \psi_1') \phi_2 \psi_3 \rangle^{(b)}, \tag{9}$$

where the dimension of the primed operators is chosen so that both sides transform in the same way.

Choose an $\alpha$ and $\mu$ on the right hand side. For example, $\alpha = +$ and $\mu = +\frac{1}{2}$. Use parity selection rules to cut down the number of terms on the right hand side. Check that there is a $C_b^a(\mu,\nu)$ which solves the crossing equation.